An improved time series approach for estimating groundwater recharge from groundwater level fluctuations

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Research Context

- Recharge: fundamental to understand for water resources management and aquifer vulnerability.
- Now easy to estimate with remote sensing data and established modelling approaches even for ‘data sparse’ areas.
- But, often don’t have the right data to evaluate the estimates or to choose between models.
- Recharge processes poorly understood & constrained in water scarce areas e.g. indirect recharge (most semi-arid to arid areas) or lateritic soils (8% of Earth land surface and much of SSA).
- GWL monitoring records potentially offer enormous insights, but getting at recharge can be tricky…
The Problem

- **Water Table Fluctuation (WTF) technique** (with normal caveats):

  \[
  q = S_y \frac{\partial h}{\partial t} + D
  \]

  - $q$ = recharge rate
  - $D$ = net groundwater drainage rate
  - $h$ = head
  - $t$ = time
  - $S_y$ = specific yield

  - $D$ is unknown or assumed hard to estimate (so too is $S_y$, but that's another story...)
  - Can we relate $D$ to aquifer parameters to create a time series method for smoothly varying WTs?

  \(\Delta h + D \cdot \Delta t = 17 \text{ cm} \)

  \[
  \text{if } S_y = 0.1 \\
  \Rightarrow \text{recharge} = 1.7 \text{ cm} \\
  \text{(rainfall in period} = 8.4 \text{ cm})
  \]
Analytical solution: (e.g. Erskine 1997)

\[ h(x,t) = \text{Re} \left[ \frac{q_a}{S \omega} \left( 1 - \frac{\cosh \lambda x}{\cosh \lambda L} \right) + \frac{q_a}{2T} \left( x^2 - L^2 \right) \right] \]

With: \( \lambda^2 = \frac{i \omega S_Y}{T} \)

Linearised Boussinesq equation:

\[ T \frac{\partial^2 h}{\partial x^2} = S_Y \frac{\partial h}{\partial t} - q(t) \]

\[ D = S_Y \frac{\partial h}{\partial t} - q = T \frac{\partial^2 h}{\partial x^2} \]

Analytical solution: (e.g. Erskine 1997)

\[ h(x,t) = \text{Re} \left[ \frac{q_a e^{i \omega t}}{S \omega} \left( 1 - \frac{\cosh \lambda x}{\cosh \lambda L} \right) + \frac{q_a}{2T} \left( x^2 - L^2 \right) \right] \]

With: \( \lambda^2 = \frac{i \omega S_Y}{T} \)

Transient component of \( h \) (m)

\[ q_a = 0.0003 \text{ m/d}, \; L=5000 \text{ m}, \; T = 250 \text{ m}^2/\text{d}, \; \omega = 2\pi/365 \]
How does $D$ vary in time and space?

$$D = S_Y \frac{\partial h}{\partial t} - q = T \frac{\partial^2 h}{\partial x^2}$$

$$D(x,t) = \text{Re} \left[ -q_a e^{i\omega t} \left( \frac{\cosh \lambda x}{\cosh \lambda L} \right) + q_a \right], \quad \lambda^2 = \frac{i \omega S_Y}{T}$$

$q_a = 0.0003 \text{ m/d}, L=5000 \text{ m}, T = 250 \text{ m}^2/\text{d}, \omega = 2\pi/365$

But, amplitude of $D$: $A = \left| q_a \left( \frac{\cosh \lambda x}{\cosh \lambda L} \right) \right|$ is small for much of many aquifers

Thus, $D \approx q_a$ is a reasonable approx. if $T/S$ and $x/L$ not too high
Time series equation for recharge

Since \( \bar{h} = \frac{q_a}{2T} \left( L^2 - x^2 \right) \)

If \( D \approx q_a \), then \( D \approx q_a = \frac{2\bar{h}T}{L^2 - x^2} \)

And \( q_i = \frac{S_Y \left( h_t - h_{(t-\Delta t)} \right)}{\Delta t} + \frac{2\bar{h}T}{(L^2 - x^2)} \)

- Can develop an equivalent equation for the non-linearised case
- The analysis holds true for non-sinusoidal recharge and for a range of other non-ideal conditions (tested with numerical models)
- Beware - \( D \) has a complex relationship with \( h \) and may be inversely proportional to \( h \) in some cases (contrary to common assumptions)
Case study from Shropshire, UK

- Geometry appropriate. Best estimates of $L = 5$ km, $S_y = 0.1$, $T = 200$ m$^2$/d.

- Monitoring wells sufficiently far from drainage outlet $A/D < 0.005$
Results for Shropshire, UK

- EA: 114 mm/a
- 2042: 110 mm/a
- 2086: 127 mm/a
NE Uganda: Location/Geological Context

Study Area

MacDonald et al (2005)
• Depth to bedrock variable: 3 to 18 m
• Regolith: sandy clay/laterite
Climate trends

Annual total rainfall and annual average air temperature derived from CRU2.1 data.
Groundwater Level Monitoring
RF Average = 1300 mm/a

PEt Average = 1900 mm/a
Soil Moisture Balance Model

- Simple ‘Penman Grindley’ type in VBA
- Daily calculations using RF and PE inputs
- Assumed runoff 0 to 5%, related to SMD/intensity
- Monthly variable $C = 43-76$ mm, $D = 74-127$ mm
- Gives ‘potential’ recharge

1-D Unsaturated Flow Model - HYDRUS

- Assumes uniform flow governed by Richards Equation
- Atmospheric boundary condition with surface run-off
- van Genuchten parameters from Rosetta for a range of soil types
- Daily stress periods using RF and PE inputs
- Feddes model for crop transpiration
- Estimates ‘actual’ recharge
Uganda

Results

10 year average recharge (mm/a)

WTF = 59 (T=5 m$^2$/d, S=1.4%) to 236 (T=20 m$^2$/d, S=5.5%)

SMBM = 164 (5% runoff) to 231 (no runoff, reduced C & D)

Hydrus = 246 (sandy clay loam)
Or, a very simple (but effective) forward model

\[ h_t = h_{(t-\Delta t)} + \frac{\Delta t}{S_y} \left( q_t - \frac{2\bar{h}T}{(L^2 - x^2)} \right) \]

\[ q_t = A \cdot P_t \]

Range of feasible parameter combinations:

- \( q = 4, \ 11 \ \text{or} \ 18\% \ \text{of rainfall} \)
- \( T = 20, \ 12.5 \ \text{or} \ 5 \ \text{m}^2/\text{d} \)
- \( S_y = 5.5, \ 3.4 \ \text{or} \ 1.4\% \)
Results & Implications

- Rainfall causes rapid WT responses (>5 m depth)
- Recharge occurs without SMD having to be overcome
  i.e. Preferential flow dominates the recharge response but more work needed to unravel processes

- Groundwater recharge not currently sensitive to changes in PE (temperature) but very sensitive to changes in rainfall amount (& intensity??)
- SMBMs and uniform flow models not good tools in such soils despite their convenience
- Emphasises need for hydraulic corroboration of recharge modelling techniques – or, if not, serious consideration of model structural error
- Recharge relatively high - changes to absolute values of recharge perhaps not as important as access/demographic pressures unless groundwater irrigation increases
- Need to know more about preferential flow processes to predict susceptibility of recharge e.g. to landuse change

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<th>SMBM</th>
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</table>
Uganda Conclusions

• Utility of a simple scoping model for testing the relationship between feasible recharge models and aquifer parameters

• Significance of preferential flow in lateritic soils:
  - Recharge less sensitive, directly, to changes in PE than may have been expected
  - Uniform flow models and SMBMs not good here
  - Fast pathways for contaminants

• Importance of sustained, high temporal resolution, groundwater level monitoring records to inform process understanding and trends

• More work needed on recharge in lateritic soils
Overall Conclusions

• Analytical simplification gives powerful insight into the relationships between recharge, aquifer parameters and WTFs

• For many parts of many aquifers ‘net groundwater drainage = average recharge’ is a good first assumption (for low GWABS)

• Method links aquifer parameters ($T$, $L$ and $S_y$) to recharge thus reducing uncertainties if these are relatively well constrained

• Can also use the analysis to forward model groundwater level fluctuations if recharge can be estimated by other means – useful for ‘conceptualisation’ stage of a water resources project

• Limitations for catchments with strong spatial trends in aquifer properties, very dynamic groundwater abstractions and/or dominated by indirect recharge

• Corroboration using multiple recharge estimates still recommended
Any questions or suggestions?

Further reading:
