An improved time series approach for estimating groundwater recharge from groundwater level fluctuations

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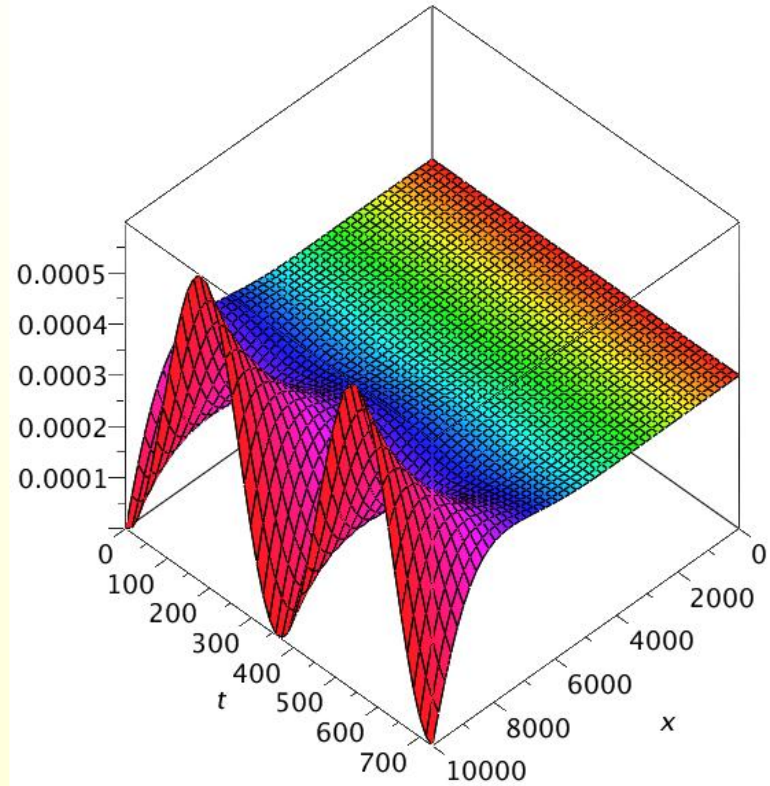

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Research Context

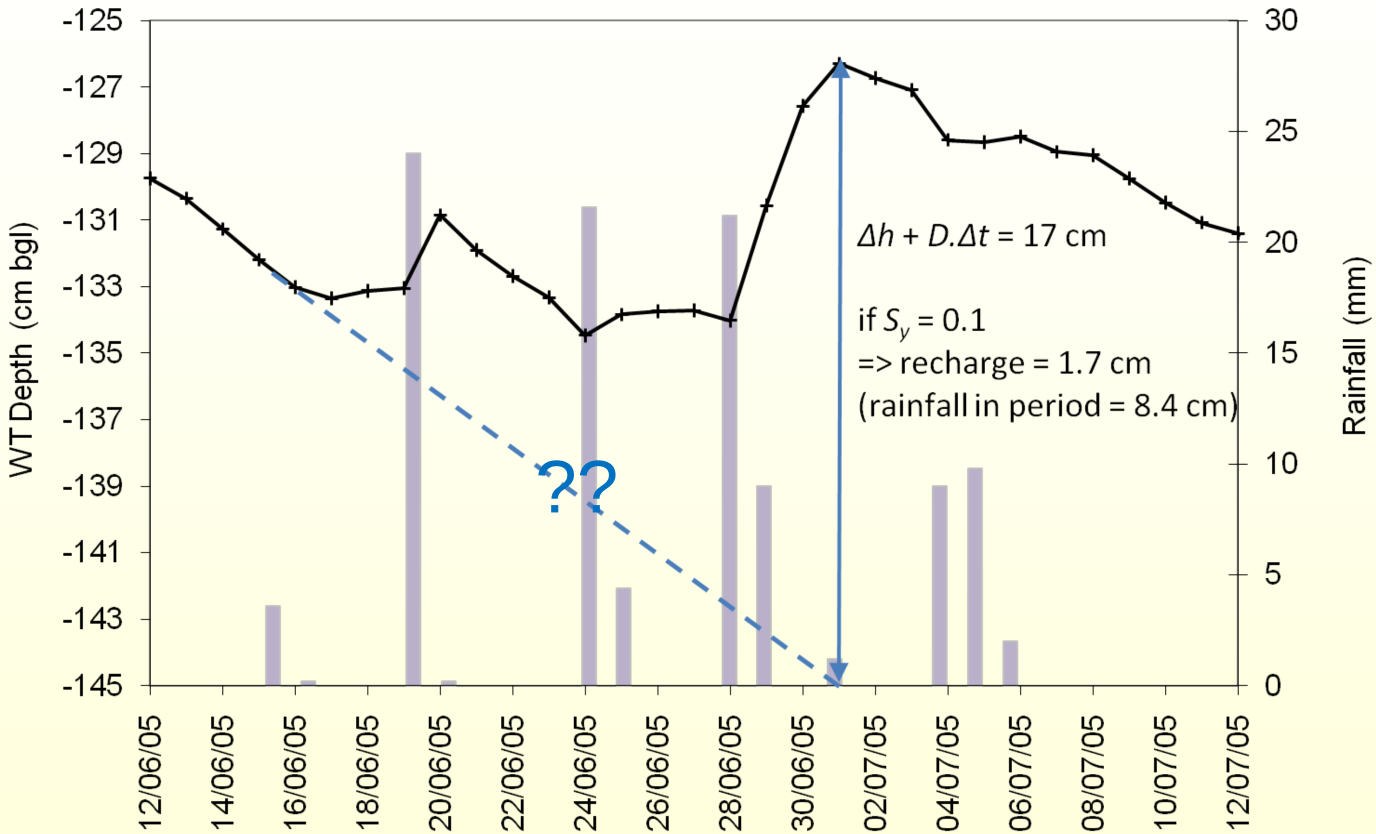
- Recharge: fundamental to understand for water resources management and aquifer vulnerability
- Now easy to estimate with remote sensing data and established modelling approaches even for ‘data sparse’ areas
- But, often don’t have the right data to evaluate the estimates or to choose between models
- Recharge processes poorly understood & constrained in water scarce areas e.g. indirect recharge (most semi-arid to arid areas) or lateritic soils (8% of Earth land surface and much of SSA)
- GWL monitoring records potentially offer enormous insights, but getting at recharge can be tricky…

The Problem

- Water Table Fluctuation (WTF) technique (with normal caveats):

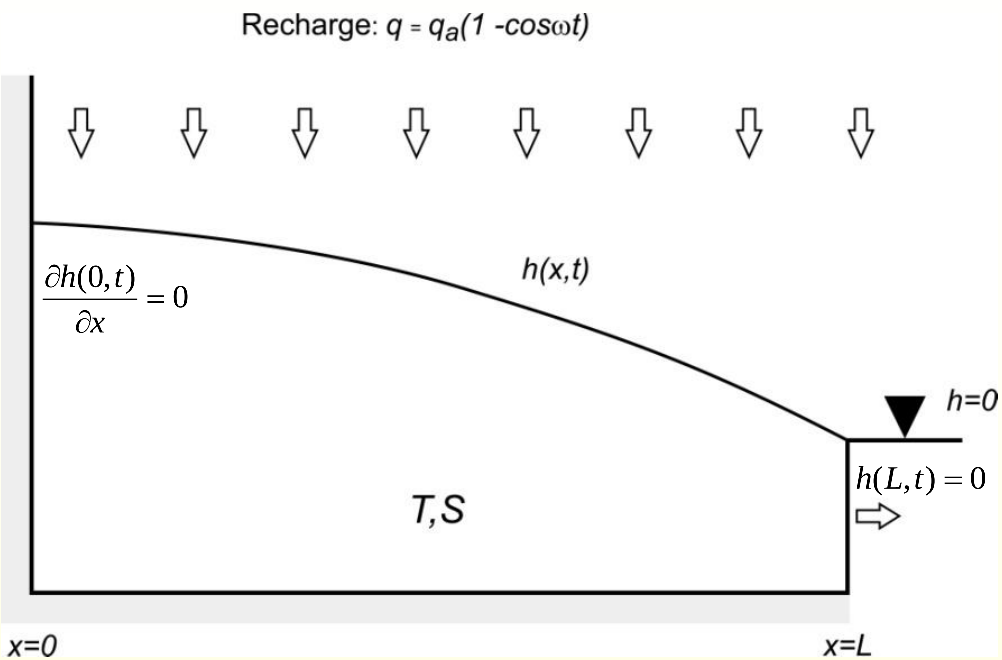
$$q = S_Y \frac{\partial h}{\partial t} + D$$

q = recharge rate
D = net groundwater drainage rate
h = head
t = time
S_y = specific yield



- D is unknown or assumed hard to estimate (so too is S_y but that's another story...)
- Can we relate D to aquifer parameters to create a time series method for smoothly varying WTs?

Theory for an 'Ideal' Aquifer

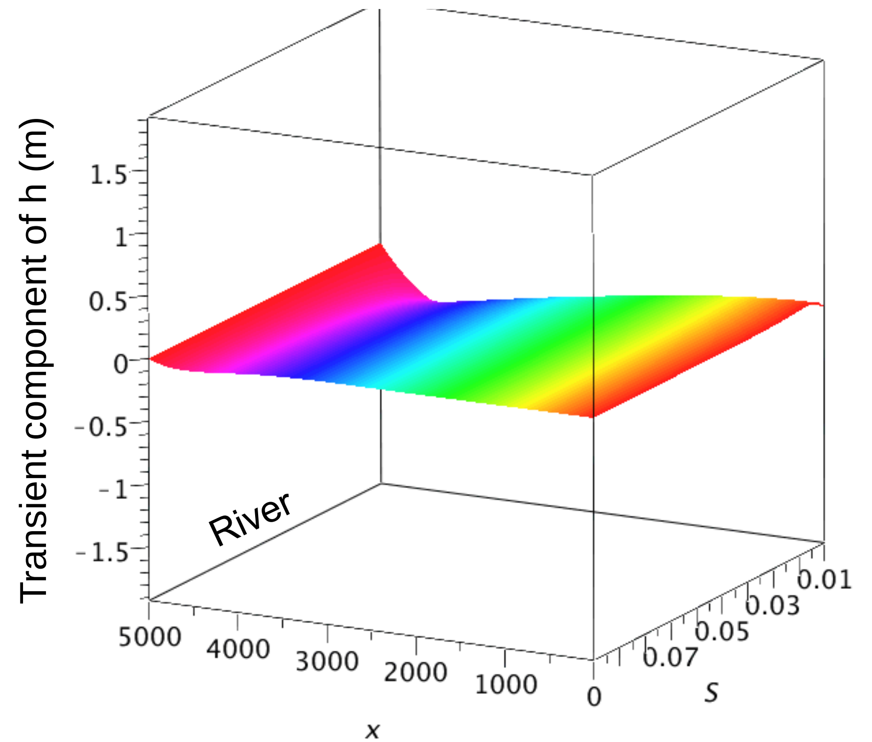


Linearised Boussinesq equation:

$$T\frac{\partial^2 h}{\partial x^2} = S_Y\frac{\partial h}{\partial t} - q(t)$$

$$D = S_Y\frac{\partial h}{\partial t} - q = T\frac{\partial^2 h}{\partial x^2}$$

q_a = 0.0003 m/d, L=5000 m, T = 250 m^2/d, ω = 2π/365



Analytical solution: (e.g. Erskine 1997)

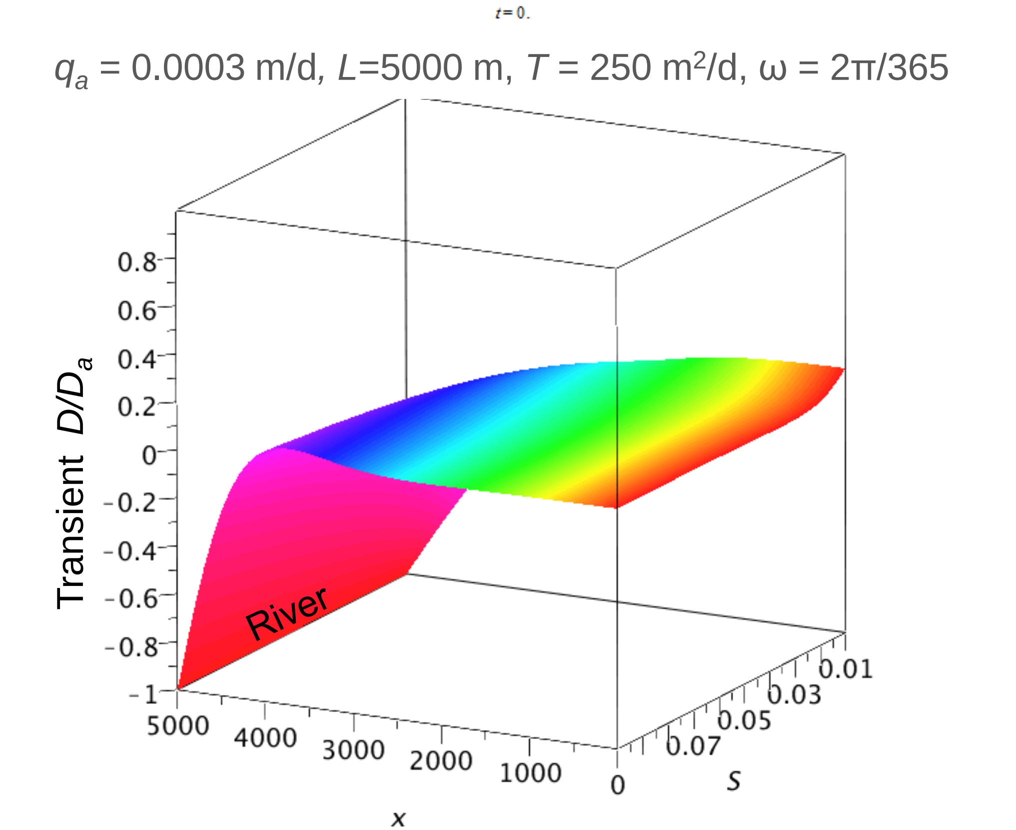
$$h(x,t) = \mathrm{Re}\left[-\frac{q_a e^{i\omega t}}{S i\omega}\left(1 - \frac{\cosh\lambda x}{\cosh\lambda L}\right) - \frac{q_a}{2T}\left(x^2 - L^2\right)\right]$$

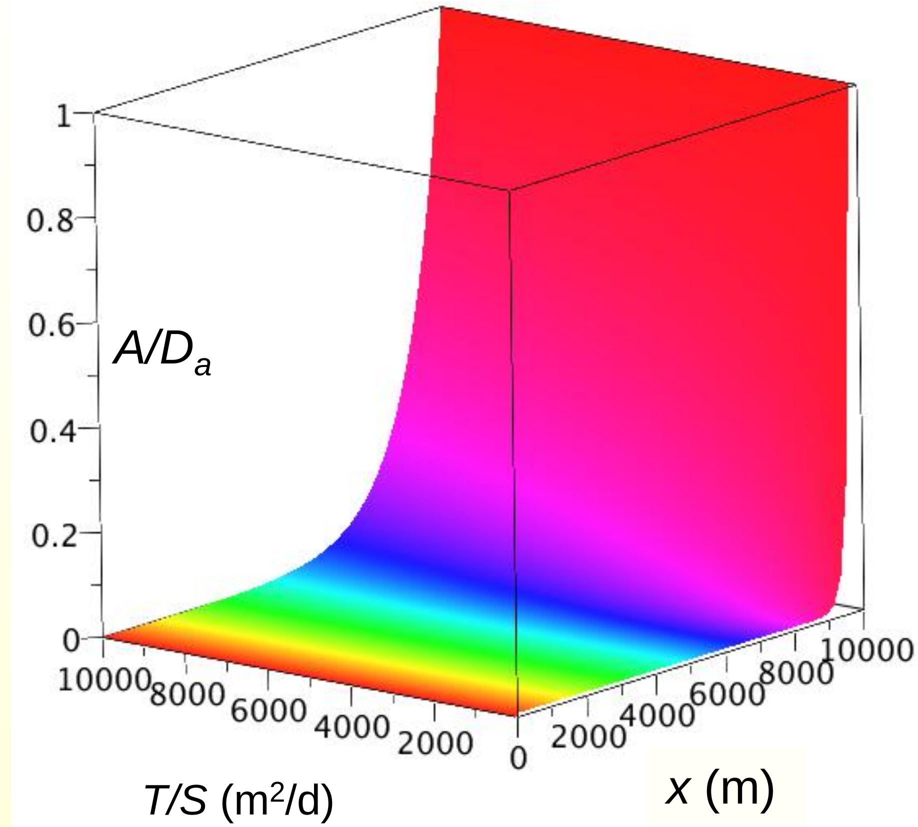
With: $\lambda^2 = \dfrac{i\omega S_Y}{T}$

How does D vary in time and space?

$$D = S_Y \frac{\partial h}{\partial t} - q = T\frac{\partial^2 h}{\partial x^2} \qquad D(x,t) = \mathrm{Re}\left[-q_a e^{i\omega t}\left(\frac{\cosh \lambda x}{\cosh \lambda L}\right) + q_a\right], \quad \lambda^2 = \frac{i\omega S_Y}{T}$$

q_a = 0.0003 m/d, L=5000 m, T = 250 m²/d, ω = 2π/365





But, amplitude of D: $A = \left| q_a \left(\frac{\cosh \lambda x}{\cosh \lambda L}\right)\right|$ is small for much of many aquifers

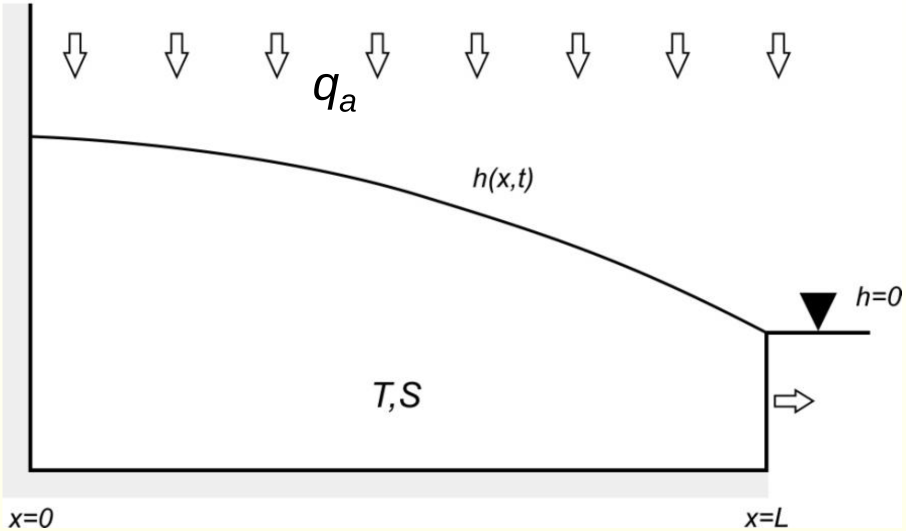
Thus, $D \approx q_a$ is a reasonable approx. if T/S and x/L not too high

Time series equation for recharge

Since $\bar{h} = \frac{q_a}{2T}\left(L^2 - x^2\right)$

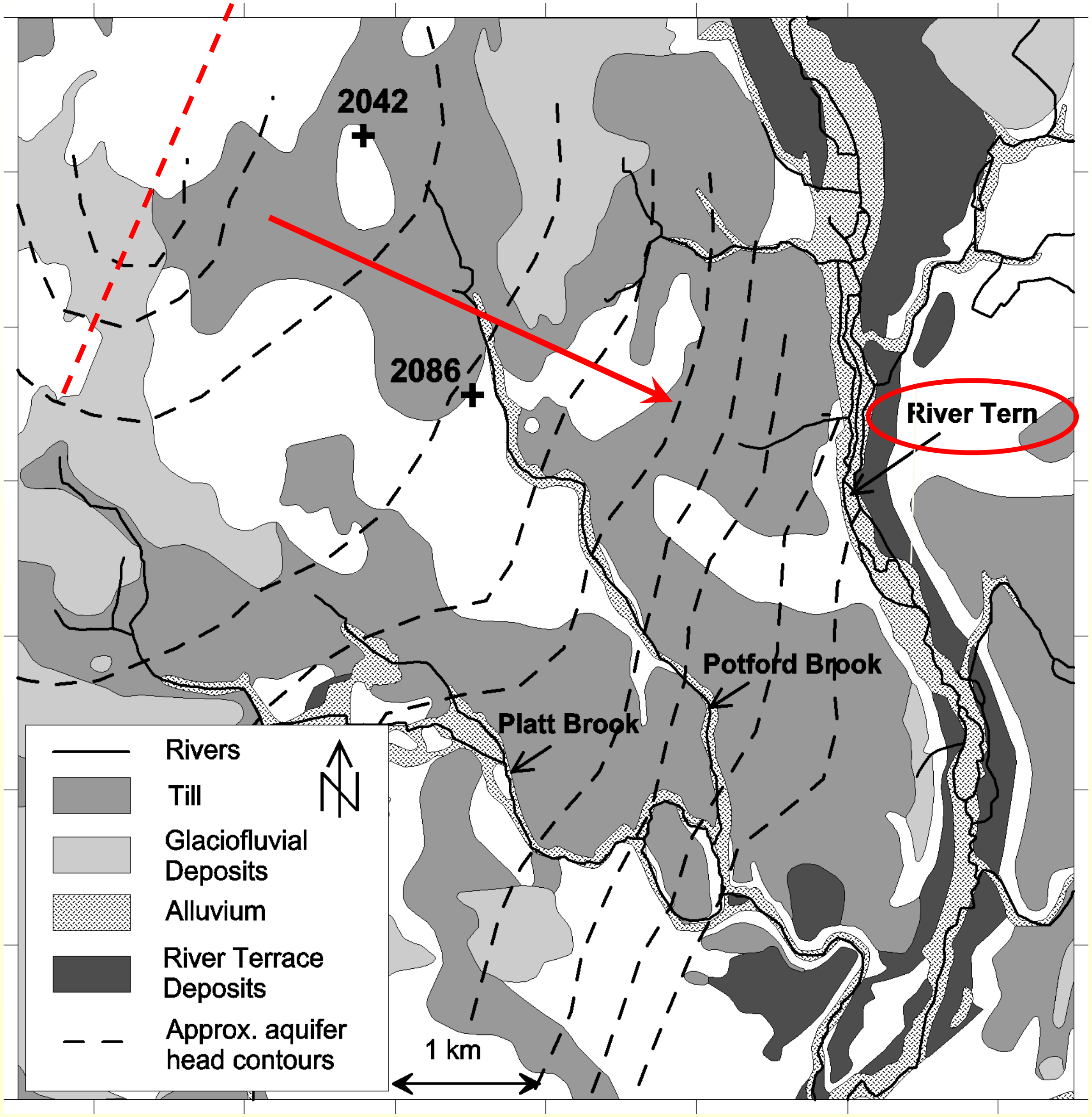
If $D \approx q_a$, then $D \approx q_a = \frac{2\bar{h}T}{L^2 - x^2}$

And $\boxed{q_t = \frac{S_Y.\left(h_t - h_{(t-\Delta t)}\right)}{\Delta t} + \frac{2\bar{h}T}{\left(L^2 - x^2\right)}}$



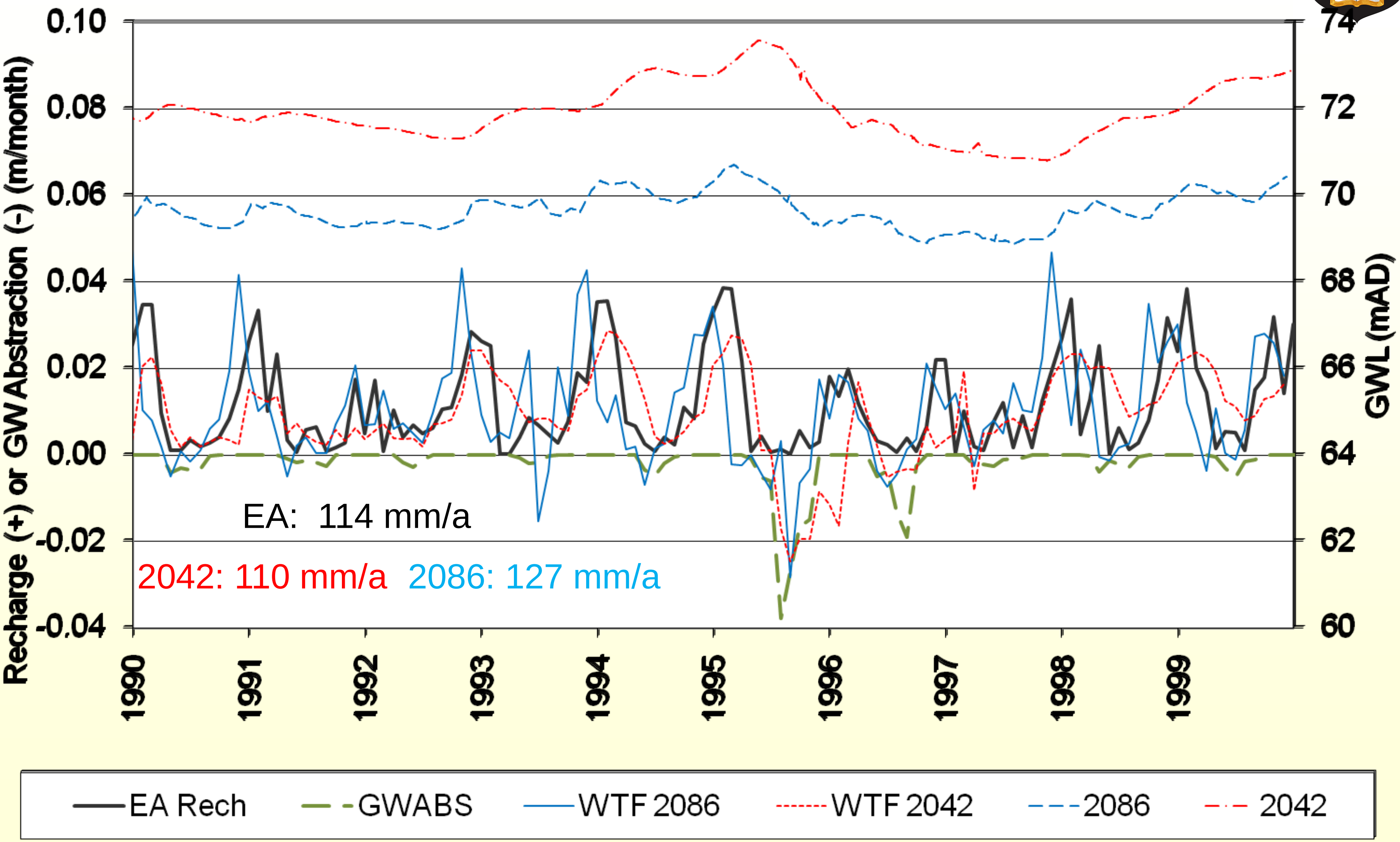
- Can develop an equivalent equation for the non-linearised case
- The analysis holds true for non-sinusoidal recharge and for a range of other non-ideal conditions (tested with numerical models)
- Beware - *D* has a complex relationship with *h* and may be inversely proportional to *h* in some cases (contrary to common assumptions)

Case study from Shropshire, UK

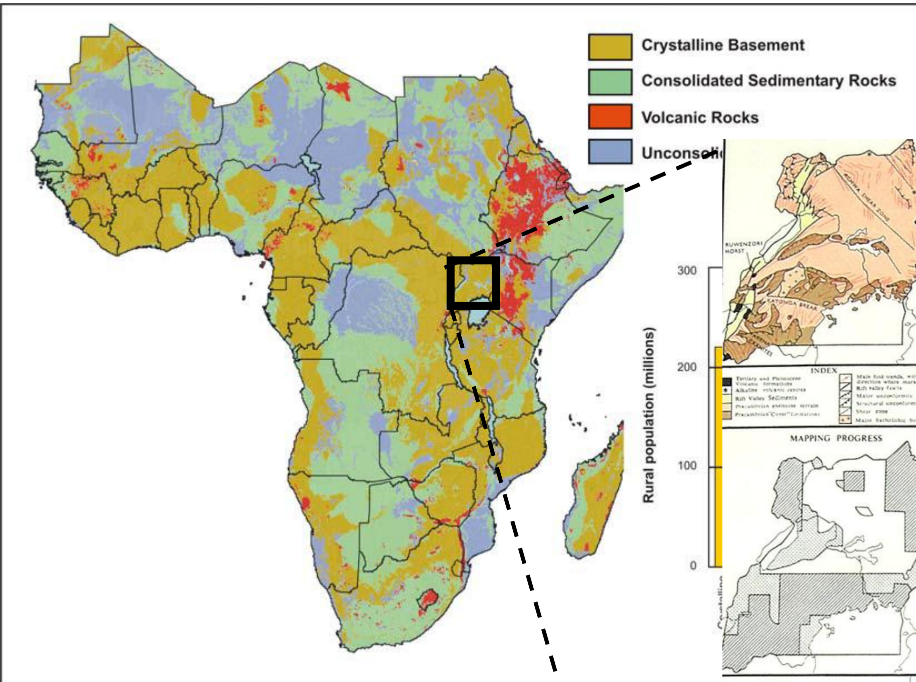


- Geometry appropriate. Best estimates of L = 5 km, S_y = 0.1, T = 200 m^2/d.
- Monitoring wells sufficiently far from drainage outlet A/D < 0.005

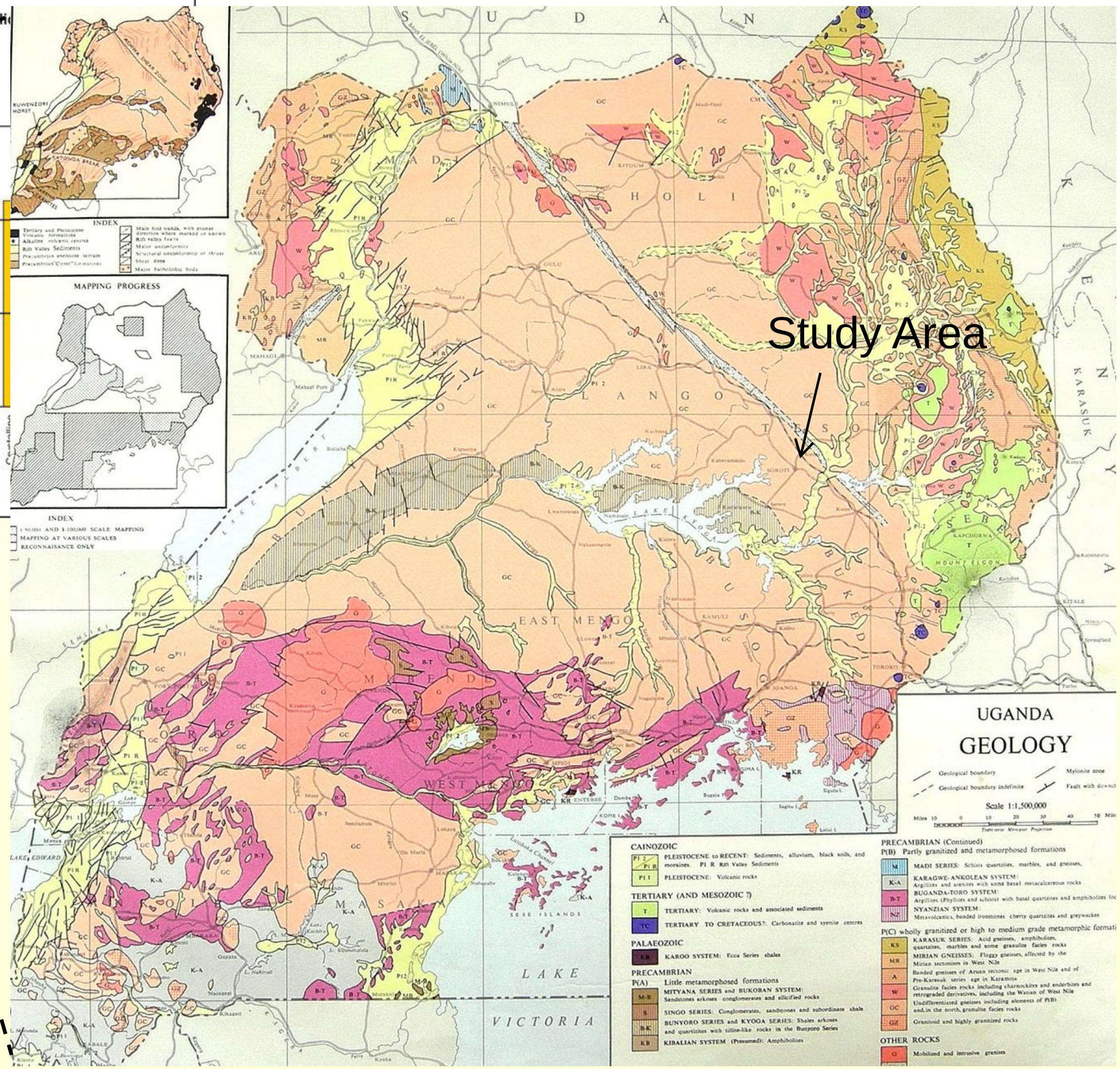
Results for Shropshire, UK



NE Uganda: Location/Geological Context

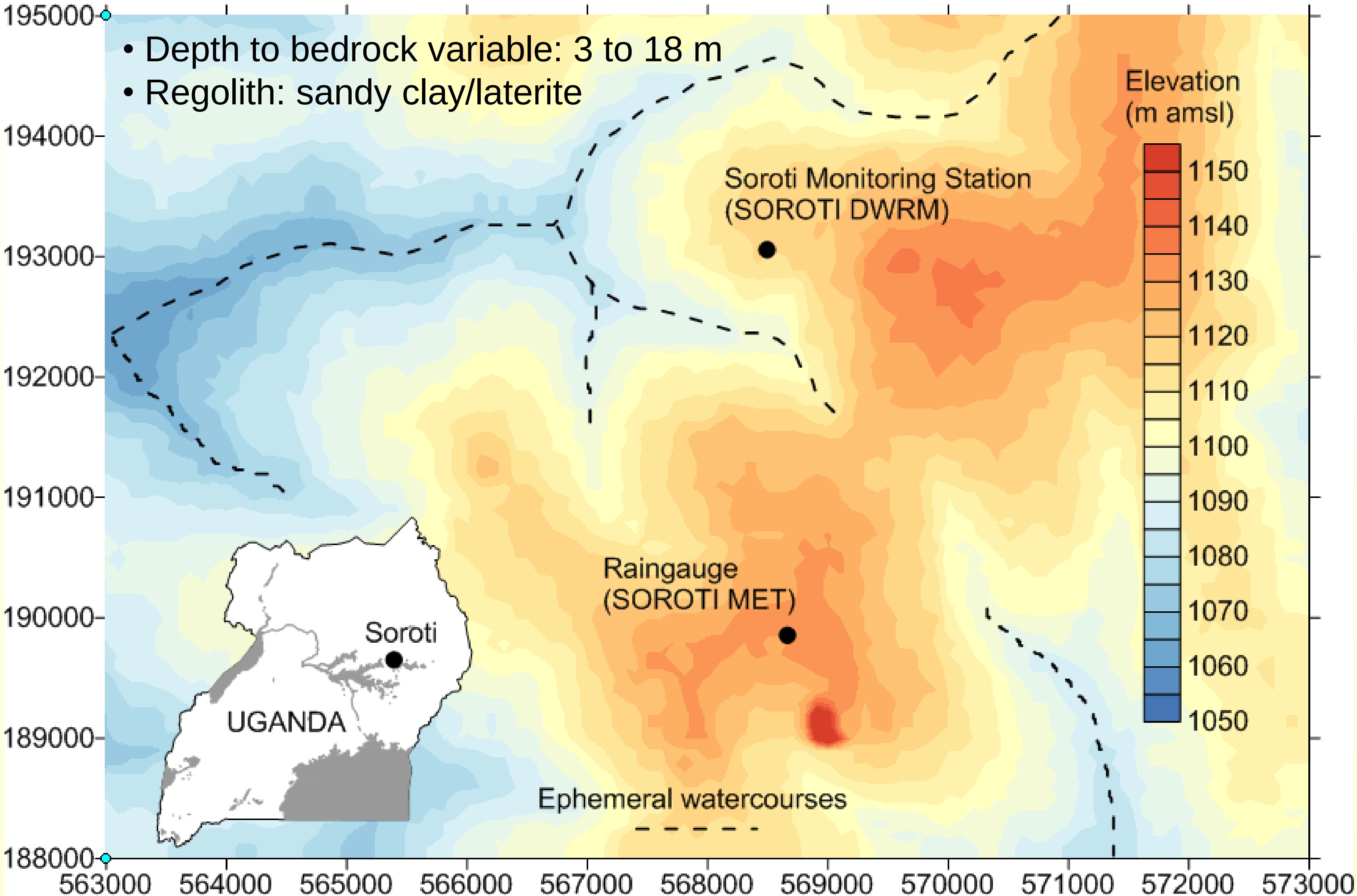


MacDonald et al (2005)

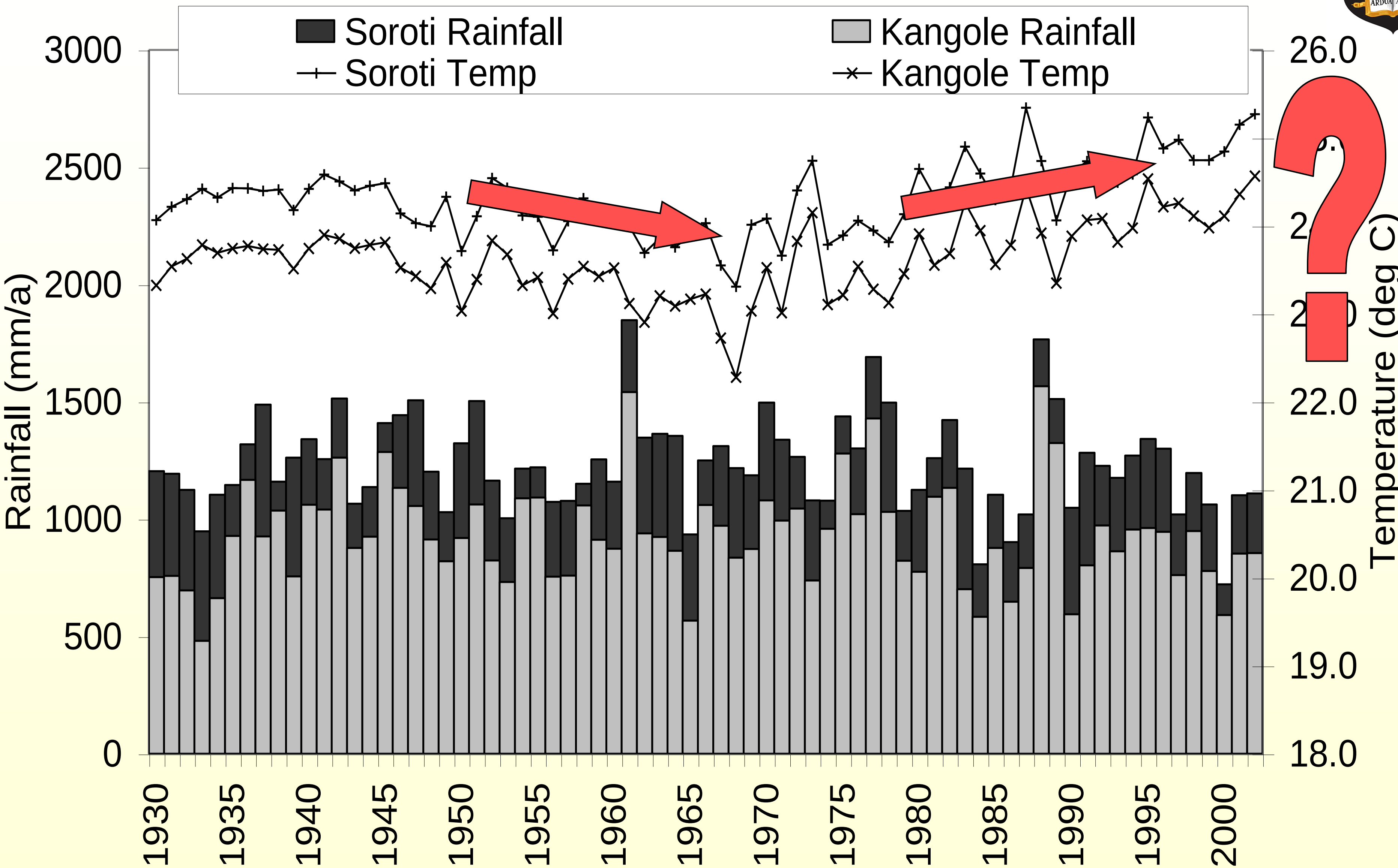


Topography & Drainage

- Depth to bedrock variable: 3 to 18 m
- Regolith: sandy clay/laterite



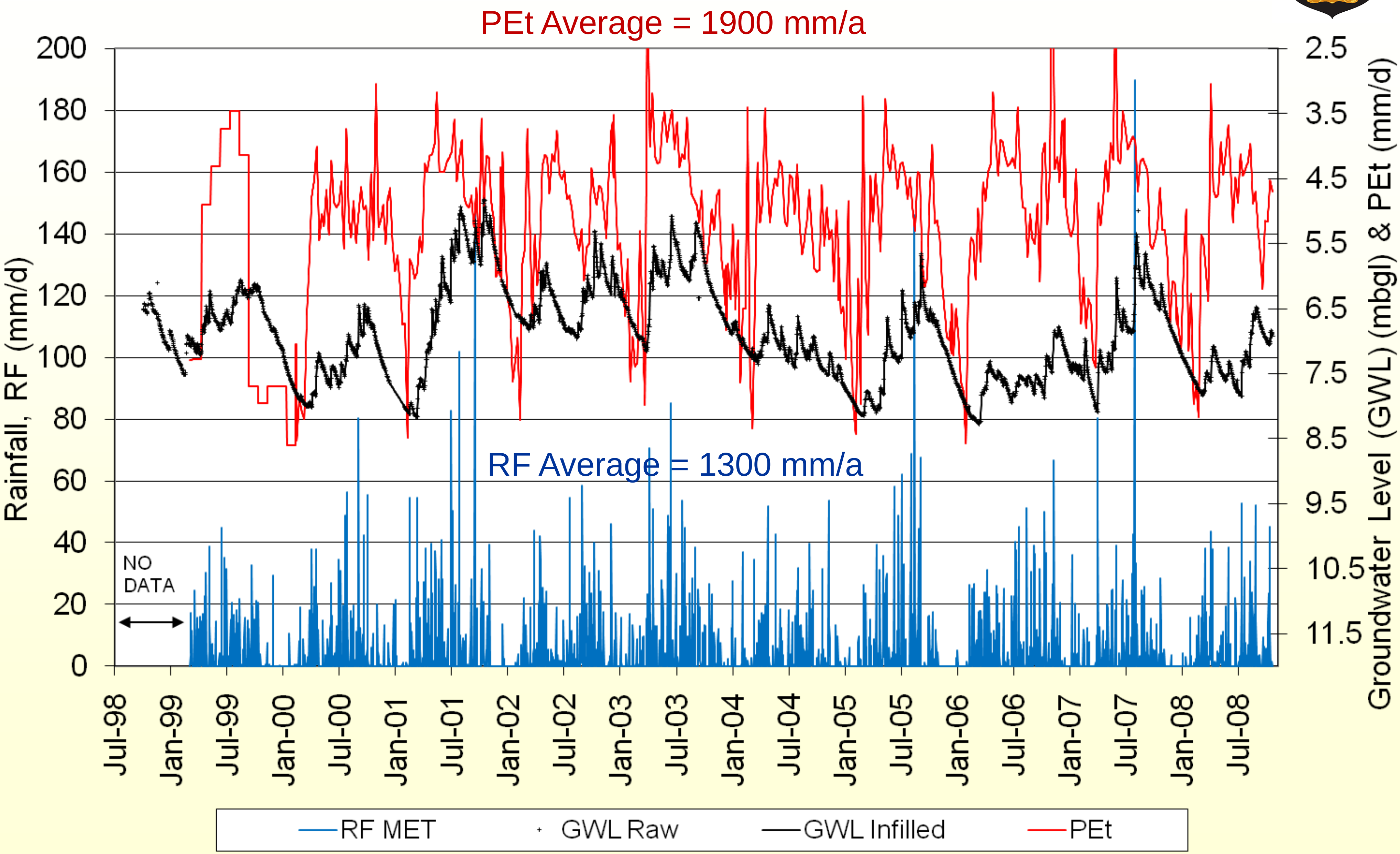
Climate trends



Annual total rainfall and annual average air temperature derived from CRU2.1 data.

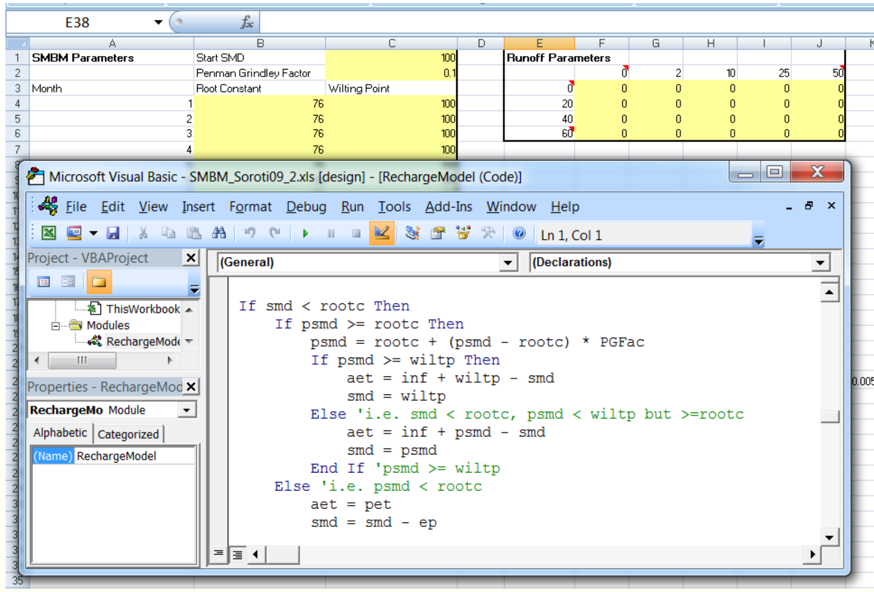
Groundwater Level Monitoring

Groundwater and Meteorological Data



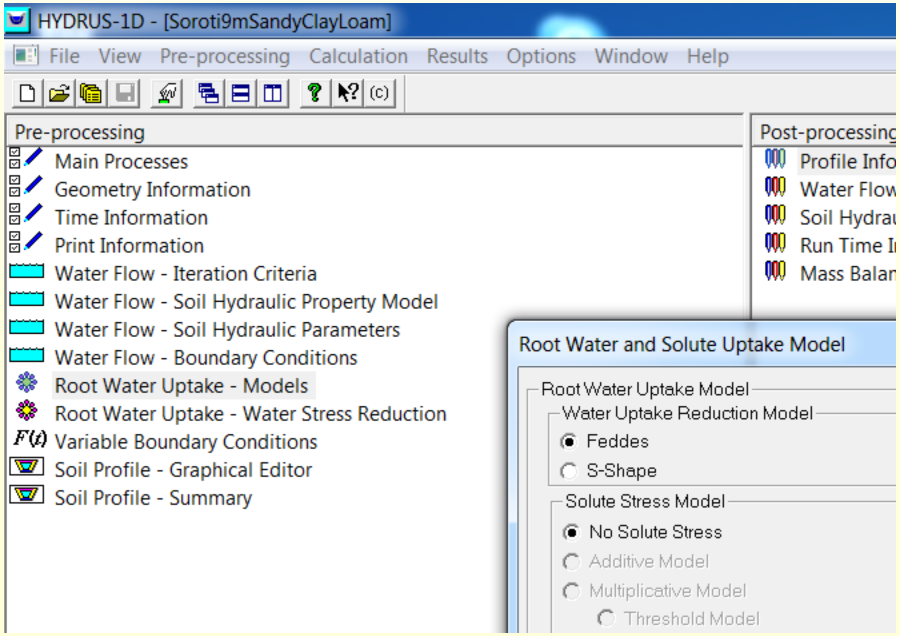
Soil Moisture Balance Model

- Simple ‘Penman Grindley’ type in VBA
- Daily calculations using RF and PE inputs
- Assumed runoff 0 to 5%, related to SMD/intensity
- Monthly variable C = 43-76 mm, D = 74-127 mm
- Gives ‘potential’ recharge



1-D Unsaturated Flow Model - HYDRUS

- Assumes uniform flow governed by Richards Equation
- Atmospheric boundary condition with surface run-off
- van Genuchten parameters from Rosetta for a range of soil types
- Daily stress periods using RF and PE inputs
- Feddes model for crop transpiration
- Estimates ‘actual’ recharge



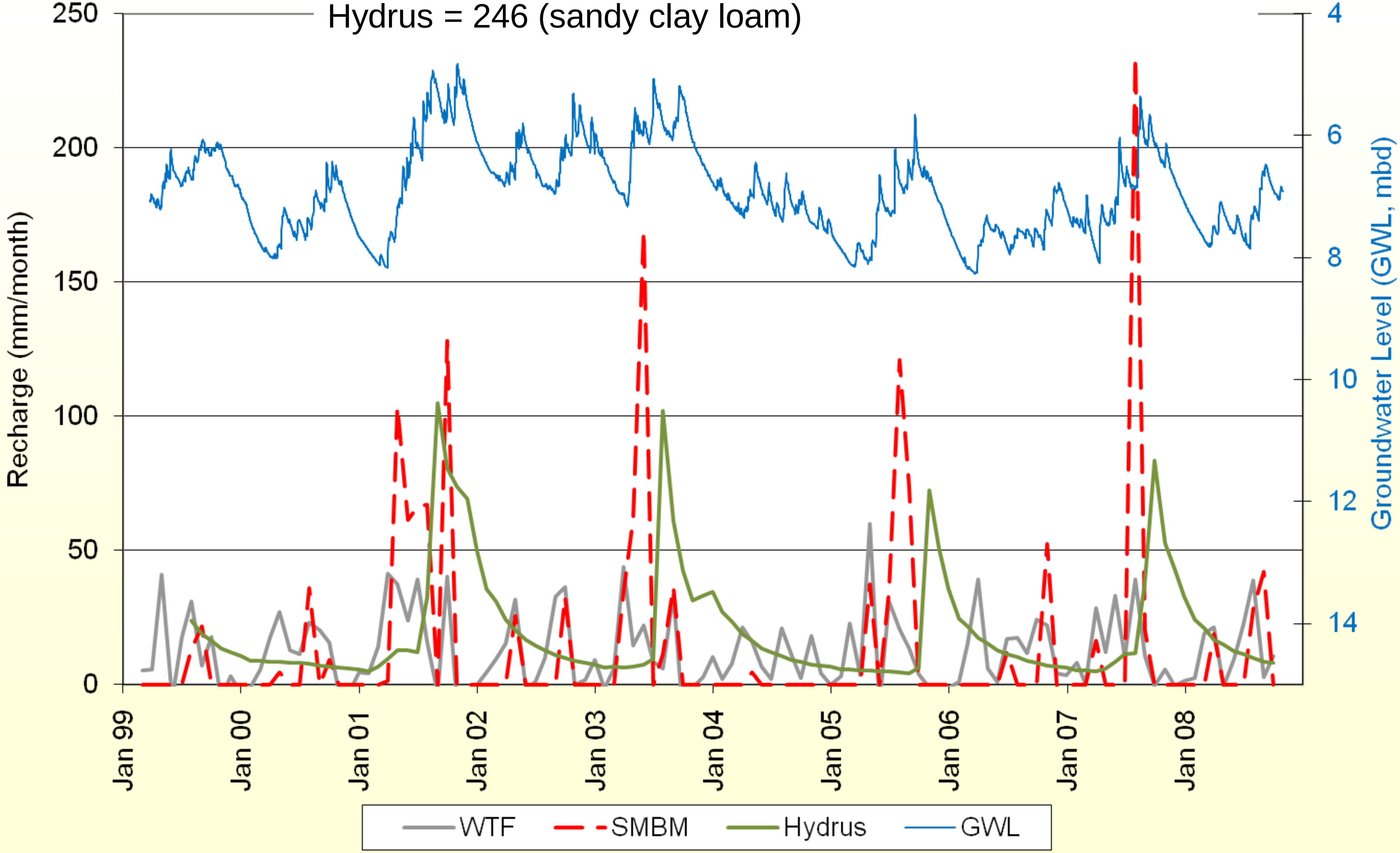
Uganda Results

10 year average recharge (mm/a)

WTF = 59 (T=5 m^2/d, S=1.4%) to 236 (T=20 m^2/d, S=5.5%)

SMBM = 164 (5%runoff) to 231 (no runoff, reduced C & D)

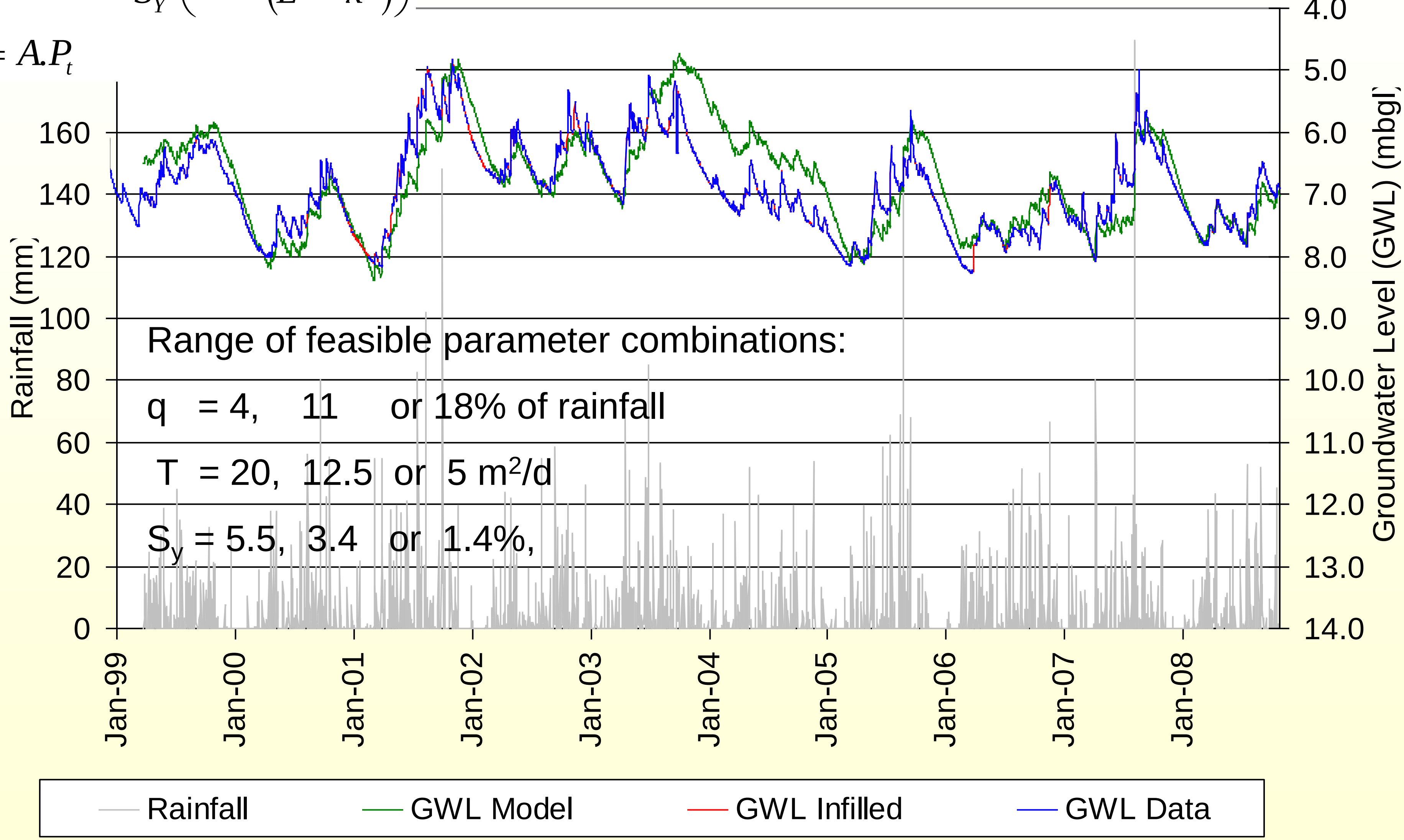
Hydrus = 246 (sandy clay loam)



Or, a very simple (but effective) forward model

$$h_t = h_{(t-\Delta t)} + \frac{\Delta t}{S_Y}\left(q_t - \frac{2\bar{h}T}{\left(L^2 - x^2\right)}\right)$$

$$q_t = A.P_t$$



Results & Implications

Year	Rainfall	PEt	WTF	SMBM	Hydrus	Raindays
2000	1165	2106	123	50	95	128
2001	1765	1819	200	427	425	144
2002	1223	1897	140	58	242	133
2003	1479	1829	131	312	320	136
2004	1055	1958	123	4	189	124
2005	1334	1977	147	267	176	97
2006	1350	1789	158	64	179	175
2007	1370	1850	150	271	277	145

- Rainfall causes rapid WT responses (>5 m depth)
- Recharge occurs without SMD having to be overcome

i.e. Preferential flow dominates the recharge response but more work needed to unravel processes

- Groundwater recharge not currently sensitive to changes in PE (temperature) but very sensitive to changes in rainfall amount (& intensity??)
- SMBMs and uniform flow models not good tools in such soils despite their convenience
- Emphasises need for hydraulic corroboration of recharge modelling techniques – or, if not, serious consideration of model structural error
- Recharge relatively high - changes to absolute values of recharge perhaps not as important as access/demographic pressures unless groundwater irrigation increases
- Need to know more about preferential flow processes to predict susceptibility of recharge e.g. to landuse change

Uganda Conclusions

- Utility of a simple scoping model for testing the relationship between feasible recharge models and aquifer parameters
- Significance of preferential flow in lateritic soils:
  - Recharge less sensitive, directly, to changes in PE than may have been expected
  - Uniform flow models and SMBMs not good here
  - Fast pathways for contaminants
- Importance of sustained, high temporal resolution, groundwater level monitoring records to inform process understanding and trends
- More work needed on recharge in lateritic soils

Overall Conclusions

- Analytical simplification gives powerful insight into the relationships between recharge, aquifer parameters and WTFs
- For many parts of many aquifers ‘net groundwater drainage = average recharge’ is a good first assumption (for low GWABS)
- Method links aquifer parameters (*T*, *L* and S_y) to recharge thus reducing uncertainties if these are relatively well constrained
- Can also use the analysis to forward model groundwater level fluctuations if recharge can be estimated by other means – useful for ‘conceptualisation’ stage of a water resources project
- Limitations for catchments with strong spatial trends in aquifer properties, very dynamic groundwater abstractions and/or dominated by indirect recharge
- Corroboration using multiple recharge estimates still recommended

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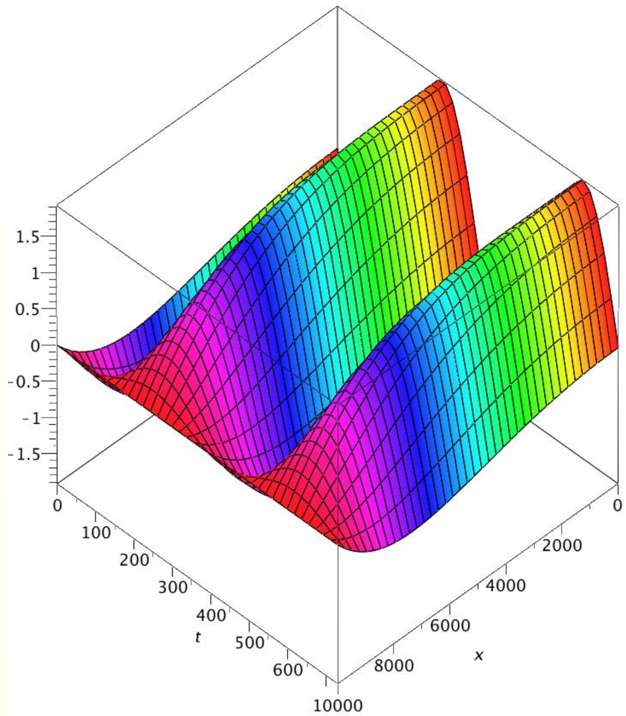
Any questions or suggestions?



Further reading:

Cuthbert, M. O. (2010).
An improved time series approach for estimating groundwater recharge from groundwater level fluctuations.
Water Resources Research, 46, W09515.

Cuthbert, M. O. & Tindimugaya, C. (2010).
The importance of preferential flow in controlling groundwater recharge in tropical Africa and implications for modelling the impact of climate change on groundwater resources.
Journal of Water and Climate Change, 1(4), 234-245.